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Portfolio Balance and the Assignment Problem in an Interdependent World

Abstract

Central to the determination of the proper assignment of monetary and fiscal policy is the precise nature of the role of international capital movements. Mundell's choice of assignments was based on his assumption that capital movements represent sustainable flows, whereas recent work in portfolio theory has shown that the demand for assets is a stock demand. Numerous papers have criticized Mundell for his specification of the capital flow equation and, instead, argued that a change in the rate of return on an asset will lead to a permanent change in desired asset stocks which can be accommodated by temporary asset flows.

Disciplines

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"Portfolio Balance and the Assignment
Problem in an Interdependent World"

by

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Mundell (1962) demonstrated that in order to achieve balance of payments equilibrium and full employment, monetary policy should be paired with external balance and fiscal policy with internal balance. The fundamental problem posed by Mundell concerned the methods governmental authorities should utilize to insure that both internal and external balance would be achieved when the underlying structural parameters of the economy were unknown. Mundell argued that the assignment of monetary policy to external balance and fiscal policy to internal balance followed from the Principle of Effective Market Classification (Mundell 1962, 76): "Policies should be paired with objectives on which they have the most direct effect."

The number of papers that have emerged from this original contribution is an indication of the interest and importance of the problem posed by Mundell. The papers which attempt to modify or extend Mundell's work do not dispute the Principle of Effective Market Classification in the two target--two instrument case,¹ but seek to determine whether fiscal policy has the most direct influence on internal balance and monetary policy on external balance. Section I of this paper discusses the problems associated with Mundell's formulation of the capital flow equation and the recent attempts to rectify Mundell's formulation. In this section we also discuss some of the logical problems encountered in dealing with the Assignment Problem for a "small country". Section II develops a two country portfolio model which views all asset demands as demands for stocks, and discusses the potential assignments for monetary and fiscal policy. Section III, which contains our principal results, discusses the stability of the various assignments and how sterilization and debt management affect stability. A concluding section summarizes our results, contrasts them with previous work and suggests directions for further research.

Section I - Introduction

Central to the determination of the proper assignment of monetary and fiscal policy is the precise nature of the role of international capital movements. Mundell's choice of assignments was based on his assumption that capital movements represent sustainable flows, whereas recent work in portfolio theory² has shown that the demand for assets is a stock demand. Numerous papers³ have criticized Mundell for his specification of the capital flow equation and, instead, argued that a change in the rate of return on an asset will lead to a permanent change in desired asset stocks which can be accommodated by temporary asset flows. It is this transitory nature of international capital flows which has led some authors, such as Levin (1972), to argue that the Mundell assignment should be reversed.

As the portfolio approach to the balance of payments has led to the recognition that capital flows are transitory in nature, it has also led to the recognition that an Official Settlements deficit or surplus is also a temporary phenomenon if the underlying system is stable.⁴ When money is viewed as an asset in individual portfolios, it follows that a continual change in the money stock--via a balance of payments deficit or surplus not fully sterilized by the monetary authorities--is inconsistent with equilibrium in a static economy. The direct implication, then, of the portfolio approach to the balance of payments is that an Official Settlements surplus will be positively related to an increase in the demand for money, and negatively related to an increase in the supply. Further, as a balance of trade deficit (surplus) represents net dissaving(saving) vis-a-vis the rest of the world, a balance of trade deficit (surplus) represents an excess supply (demand) for wealth.⁵ The relationships between money market disequilibria and the balance of payments have recently been explored

by several papers concerned with the monetary theory of the balance of payments,⁶ yet the relationships between the balance of trade and the excess demand for wealth have not been adequately explored.

A distinction in the literature has been made between the continuing flow effects versus stock shift effects,⁷ or what alternatively has been called the scale effect versus the composition effect.⁸ The essential distinction is that asset holders have two decisions to make concerning their portfolios: the scale effect is concerned with the determinants of desired portfolio size, while the composition effect is concerned with the determinants of the asset mix within a portfolio of a given size. As the determinants of the scale and composition decisions are not mutually exclusive, a change in the determinants of asset demands can normally be expected to produce asset flows to accommodate both the realignment of the mix of assets in a portfolio and the change in desired portfolio size.

Several authors have developed models in which the demand for assets at a point in time (composition effect) are demands for stocks while portfolio size is held constant throughout the analysis.⁹ Tsiang (1975) developed a model in which international capital movements represent a stock adjustment process in order to examine the importance of capital flows in determining the proper assignment. However, he assumes that savings, which represents a flow component of wealth, is a constant fraction of income. In an economy without population or income growth, it is quite paradoxical to state that the demands for the individual components of wealth are demands for stocks but that individuals will accumulate these stocks continually.¹⁰ By allowing asset holders to continually accumulate assets, Tsiang finds--as did Mundell--that interest rate changes will produce sustainable capital flows.

Thus, one purpose of this paper is to develop a portfolio model in which the individual asset demands, as well as the demand for wealth, represent stock decisions. The nature of the model, then, is such that if the underlying structure of the economic system is stable, equilibrium will be characterized by both balance of payments and balance of trade equilibrium. As full stock equilibrium requires balance of trade and payments equilibrium, Mundell's original question must be reformulated as the government need not assign any of its policy tools to the balance of payments if the economic system is stable. Our paper, then, attempts to answer the following two questions:

- 1) When the possibility of instability exists, "What government policies will assure convergence towards equilibrium?"
- 2) When the balance of payments is self-correcting, or when the government pursues an assignment which guarantees stability, "What policies act to increase the speed of adjustment?"

The distinction between portfolio size and composition effects suggests that it is important to examine both the wealth creation effects of policy instruments and the particular form in which they create wealth. For any assignment of fiscal policy, we then determine whether the change in government expenditures should be financed by changes in taxes, increases in the stock of bonds or increases in the money stock.¹¹

We propose to study the Assignment Problem in a two country model. The usual small country assumption greatly simplifies economic analysis at the expense of relevance for large economic regions such as the U.S., EEC or U.K. Further, the properties of the "small country models" differ from those of large countries. McKinnon (1968) shows that in a small country portfolio model, fiscal policy has no permanent effects on the level of income unless capital is immobile and the exchange rate is flexible (i.e.--unless the small country is effectively closed). Open market

operations, on the other hand, can only alter the level of income if the exchange rate is flexible.¹² The attempt to develop a portfolio model for a small country which is capable of dealing with the Assignment Problem is then futile in a world of perfectly fixed exchange rates--the case in which the balance of payments is a policy problem--since neither traditional monetary nor fiscal policy can alter the level of income, and since the balance of payments (if the system is stable) is self-correcting.

Section II - The Basic Model

The model postulates two countries--e.g. - the U.S. and the U.K.--in which only two assets are held (money and bonds). Residents of a country are assumed to hold only that country's money,¹³ whereas foreign bonds can be held by domestics. It is assumed that the exchange rate is kept permanently fixed and that capital markets are sufficiently integrated such that asset holders are indifferent to holding domestic or foreign bonds. Thus, there is a single world interest rate. In accord with the McKinnon (1968) and Argy and Kouri (1974) monetary models of the balance of payments, the Keynesian assumption of fixed commodity prices and variable income levels is made.¹⁴

The U.S. private sector's demands for cash balances and bond holdings are given by equations 1 and 2. These demands are functions of the current level of U.S. disposable income, the real (equal to the nominal) rate of return on bonds, and U.S. private sector wealth, ie. -

$$1) M^D = L(Y_d, r, W)$$

Where: M^D = private U.S. demand for cash balances

$$2) B^D = B^D(Y_d, r, W)$$

Y_d = U.S. disposable income

and:

r = rate of return on bonds

$$3) W = B^P + M$$

W = U.S. private sector wealth

B^D = private U.S. demand for bonds

B^P = bond holdings of the U.S.
private sector

M = money holdings of the U.S.
private sector

At a moment in time, in which wealth is fixed, the balance sheet constraint imposes certain sign restrictions on the asset demand functions. In particular, as long as wealth is fixed, the sum of the asset demands must always be equal to the given stock of wealth since it is impossible to allocate more assets than the existing stock. The above condition will be met if the sum of the effects of changes in the interest rate and changes in the level of income both sum to zero across the portfolio, while the effect of a change in wealth sums to unity across the portfolio, i.e., $\frac{\partial L}{\partial Y_d} + \frac{\partial B^D}{\partial Y_d} = \frac{\partial L}{\partial r} + \frac{\partial B^D}{\partial r} = 0$; and

$$\frac{\partial L}{\partial W} + \frac{\partial B^D}{\partial W} = 1. \text{ By assumption: } \frac{\partial L}{\partial Y_d} > 0; \frac{\partial L}{\partial r} < 0; \text{ and } 0 < \frac{\partial L}{\partial W} < 1.$$

Similarly the U.K. demands for money and bonds can be represented by:

$$4) M'^D = L'(Y'_d, r, W')$$

Where: Primed symbols represent the U.K.
counterpart of the U.S. variable.

$$5) B'^D = B'^D(Y'_d, r, W')$$

$$6) W' = B'^P + M'$$

It is assumed that money has no backing, but the rules of the game are such that there is a reserve asset in which international payments are made. When a resident of a country receives the reserve asset, the central bank immediately exchanges the reserve asset for the domestic currency. Thus, one component of each country's money supply is the cumulated sum--either positive or negative--of the central bank's accumulations of the reserve asset, each times the currency price of the reserve asset. If the currency

price of the reserve asset in both the U.S. and the U.K. is set equal to unity (necessitating an exchange rate equal to one), a component of each country's money supply is equal to the central bank holdings' of the reserve asset. The second component of a country's money supply is equal to the cumulated sum of bonds purchased by the central bank since central banks are assumed to purchase bonds only during open market operations. Thus, the money supply in each country can be represented by:

$$7) M = B^C + R$$

Where: B^C = cumulated sum of U.S. central bank bond purchases

$$8) M' = B'^C + R'$$

R = dollar value of U.S. central bank holdings of the reserve asset.

Since the world stock of the reserve asset is assumed fixed:

$$9) dR = -dR'$$

Each government is assumed to issue a fixed price bond, and since the two bonds are viewed as perfect substitutes, bond market equilibrium requires:

$$10) B + B' = B^P + B'^P + B^C + B'^C \quad \text{Where: } B = \text{stock of bonds issued by the U.S. government}$$

B' = stock of bonds issued by the U.K. government

The asset demand equations (equations 1-6) describe the demands for assets at a point in time wherein portfolio size is fixed. Over time, however, the size of a portfolio need not be constant and, following Jones (1968), it is assumed that saving is proportional to the discrepancy between desired and actual wealth. Since the desired or target level of wealth is positively related to both the level of disposable income and the interest rate, saving behavior can be represented by:

$$11) \dot{W} = \alpha[W^*(Y_d, r) - W]$$

Where: W^* = desired wealth

$s = \alpha \frac{\partial W^*}{\partial Y_d}$ = marginal propensity to save

α = constant of proportionality

A dot appearing over a variable represents the time derivative of that variable

$$12) \dot{W}' = \alpha' [W'^*(Y_d', r) - W] \quad \text{and:} \quad \frac{\partial W^*}{\partial Y_d} > 0; \quad \frac{\partial W^*}{\partial r} > 0; \quad s < 1.$$

With saving behavior specified, the consumption or expenditure function becomes a redundant equation, i.e.-

$$13) E = Y_d - \dot{W} \quad \text{Where: } E = \text{U.S. expenditures on the U.S. and U.K. good}$$

$$14) E' = Y_d' - \dot{W}'$$

Total consumption expenditures, by definition, sum to the demand for the domestic good plus the demand for the foreign good. Given fixed commodity prices and a fixed exchanged rate, the private demand for imports is solely a function of private expenditures. The total demand for imports is the sum of the private and government demands. If the private and government marginal propensities to import are equal¹⁵ total imports are solely a function of private plus government expenditures. The balance of payments equation condition states the change in the U.S. money supply due to the balance of payments is equal to the difference between U.S. exports and imports, plus net U.S. bond sales to the U.K.,¹⁶ i.e.-

$$15) \dot{R} = X(E' + G') - X'(E + G) + \dot{B} - \dot{B}^c - \dot{B}^p \quad \text{Where: } G = \text{U.S. government expenditures}$$

$X = \text{U.S. exports} = \text{U.K. imports}$

$$\frac{\partial X}{\partial (E' + G')} \equiv m' = \text{U.K. marginal propensity to import}$$

$$\frac{\partial X'}{\partial (E + G)} \equiv m = \text{U.S. marginal propensity to import}$$

$$\text{and: } 0 < m < 1; \quad 0 < m' < 1$$

Government Behavior

As each government is assumed to finance a discrepancy between its expenditures and revenues by bond issuance, the government budget constraints become:

$$16) G - T = \dot{B}$$

Where: T = U.S. tax revenues

$$17) G' - T' = \dot{B}'$$

Governments may desire to increase taxes by some porportion of any increase in government expenditures; this behavior can be represented by:

$$18) T = tG$$

Where: t = constant of proportionality
 $0 \leq t \leq 1$

$$19) T' = t'G'$$

Fiscal policy involves determining the level of government expenditures and the constant of proportionality between taxes and expenditures.

As in the Mundell model, the government can assign its deficit to internal balance or to the balance of payments. An alternative assignment, however, is to increase the government deficit when the country experiences a balance of trade surplus. As pointed out in Section I, a balance of trade surplus represents desired net saving. Since the government deficit involves wealth creation, the government may decide to assign fiscal policy to the balance of trade. The potential assignments of fiscal policy can be represented by:¹⁷

$$20) (1 - t)G = g_1 BP + g_2 BT \quad \text{Where: } BP = \text{U.S. balance of payments} = \dot{R}$$

$$+ g_3 (Y^* - Y)$$

$$BT = \text{U.S. balance of trade} = X - X'$$

$$21) (1 - t')G' = g_1' BP' + g_2' BT'$$

$$Y = \text{U.S. income level} = Y_d + T$$

$$+ g_3' (Y'^* - Y')$$

$$Y^* = \text{desired income level}$$

$$g_i = \text{positive constant } (i = 1, \dots, 3)$$

Note: $BT = -BT'$ and $BP = -BP'$

The policy instrument available to the monetary authorities is their bond holdings, which can be assigned to internal balance, the balance of payments or the balance of trade. The central bank may also attempt to monetize the government deficit by purchasing a portion of the new bond issuances. Thus, the potential assignments of monetary policy can be represented by:

$$22) \dot{B}^C = m_1 BP + m_2 BT + m_3 [Y^* - Y] + m_4 \dot{B}$$

Where: m_1 = positive or negative constant as the monetary authorities may sterilize or accommodate the balance of payments. If they sterilize $-1 < m_1 < 0$. If they accommodate $m_1 > 0$

$$23) \dot{B}'^C = m_1' BP' + m_2' BT' + m_3' [Y'^* - Y'] + m_4' \dot{B}'$$

m_2 & m_3 = positive constants

m_4 = positive constant where $0 \leq m_4 \leq 1$. Note: increases in m_4 represent increases in the amount of the government deficit financed by money issuance.

In the absence of policy active governments (i.e., $g_i = g_i' = m_i = m_i' = 0$), the world stock of money and world stock of bonds are fixed. Full stock equilibrium requires that desired portfolio size and composition equal the actual amount and distribution of assets. Portfolio size within a country will be constant if the balance of trade is zero; with a zero trade balance, portfolio composition will be invariant over time if the balance of payments equals zero. In Enders (1975) it is shown that the system is unambiguously stable if $1 - m - m' > 0$, $\frac{\partial L}{\partial r} + \frac{\partial L}{\partial W} \frac{\partial W^*}{\partial r} < 0$, and $\frac{\partial L'}{\partial r} + \frac{\partial L'}{\partial W'} \frac{\partial W'}{\partial r} < 0$; i.e.- the sum of the marginal propensities to import is less than unity and the long run demand for money is negatively related to the interest rate. Given these two standard assumptions,¹⁸ the system will converge to full stock equilibrium, although income levels need not be

the full employment levels of income. As shown in the Appendix, when the parametric values of corresponding U.S. and U.K. demands are equal, the system is unambiguously stable. These results are not surprising since, with constant levels of wealth, the asset flows operate as in the early specie-flow models. In the absence of flexible prices, however, income levels need not be consistent with full employment.

Section III - Policy Assignments and Stability

Since--in the absence of government intervention--the equilibrium levels of income attained may not be consistent with full employment, income levels become targets to which governments may assign monetary or fiscal policy. Governments, however, not only desire to promote full employment but also desire to minimize the time needed to attain full employment. Thus, we plan to determine how various assignments affect stability, and how--for a given assignment--various policy responses affect the speed of adjustment.¹⁹

The potential assignments are given by equations 20-23; it is possible to solve equations 1-23, obtain the characteristic roots of the system and discuss how each g_i and m_i affects stability. We choose, however, to follow a simpler course and first determine the stability properties of pure assignments (in which each policy is assigned to only one target). From the pure assignment case we then ascertain the effects of mixed assignments. Even when pure assignments are considered, the characteristic roots of the system are quite complicated expressions. The characteristic roots, however, can be greatly simplified if it is assumed that the values of corresponding government policy parameters are identical (i.e., $g_i = g_i^!$ and $m_i = m_i^!$). In order to simplify our presentation, the discussion in the

text of this section--unless otherwise noted--assumes that corresponding government policy parameters are identical, while the effects of asymmetrical policy parameters are relegated to footnotes. Also, the discussion in the text and footnotes assumes that the parametric values in the U.S. and U.K. asset and import demand equations are identical.

Case I - Fiscal Policy to Internal Balance, Monetary Policy to the Balance of Payments

The first assignment under consideration is the one recommended in Mundell (1962). In terms of equations 20-23, this assignment implies:

$$20a) (1-t)G = g_3(Y^* - Y); \quad 21a) (1-t')G' = g_3'(Y'^* - Y')$$

$$22a) \dot{B}^C = m_1 BP + m_4 \dot{B}; \quad 23a) \dot{B}'^C = m_1' BP' + m_4' \dot{B}'$$

As can be determined from the appendix, the characteristic roots of the dynamic system for this assignment are:²⁰

$$24) \sigma_1 = -\alpha[g_3(1-t)(1-2m) + 2m] \cdot [g_3(1-t)(1-2m) + 2m + s(1-2m)(1+tg_3)]^{-1}$$

$$\sigma_2 = -\alpha(1-t)g_3[W_r(m_4 - L_w) - L_r] \cdot [\alpha W_r L_y(1+tg_3) - L_r(g_3(1-t) + s(1+tg_3))]^{-1}$$

If $m_4 = 0$ (deficit financed through bond issuance), then sufficient conditions for stability are: a) $1 - 2m > 0$; b) $L_r + L_w W_r < 0$; and c) $t < 1$, i.e.- if the sum of the marginal propensities to import are less than unity, the long run demand for money is negatively related to the interest rate and if governments do not maintain balanced budgets. Conditions a) and b) are standard, and can be expected to be fulfilled. If $t = 1$, $\sigma_2 = 0$ and income levels do not converge to full employment. This result

follows from viewing insufficient aggregate demand as being caused by a deficiency in total wealth stocks. That is, at full employment, an insufficient level of wealth will cause a reduction in total expenditures and a corresponding reduction in income levels. Incomes will fall until the demand for wealth is equal to the stock of wealth. Thus, changes in government expenditures--unless permanently maintained--not accompanied by wealth changes will not promote full employment. From equation 25 it is also possible to determine how changes in the policy parameters influence the speeds of adjustment. As is apparent, the more responsive is fiscal policy (i.e., the larger is g_3) the faster is the speed of adjustment; and the speed of adjustment is negatively related to the tax rate.²¹

Although government expenditures undertaken to stabilize the economy should--as in a Keynesian world--be financed by wealth creation, it is possible to inquire whether the government deficits should be financed by money or bond creation. In considering the effects of changing m_4 (the extent to which debt is monetized), it is apparent that one characteristic root (σ_1) is independent of m_4 , while the absolute value of the other root (σ_2) varies directly with m_4 . Since σ_1 represents the speed of convergence of the balance of trade and payments (see footnote 21), debt management only affects the speed of adjustment of income levels. Upon reflection, this result should be obvious as the balance of trade represents desired net saving or dissaving and monetizing the debt alters only the form of wealth creation. Monetizing the debt will, however, affect the speed at which income levels converge since, as in Keynesian models, monetizing the government debt is more expansionary than financing government expenditures through bond issuance. In our model monetizing the government debt is expansionary, as it induces a lower interest rate, leading to decreases in desired wealth, and increases in expenditures.

Finally, consider the effect--or lack of effect--of monetary policy to the Balance of Payments. As seen from (24), as long as full sterilization does not occur, the degree of sterilization or accommodation of the Balance of Payments has no effect on stability or speeds of convergence. Thus, contrary to most received doctrine, it is not true that sterilization is destabilizing.²²

Although the results obtained above are predicated upon the governments adopting the same policy response parameters, similar results hold in other cases. For example, if sterilization rates are the same ($m_1 = m'_1$), while the degree of monetization and the fiscal policy response parameters differ, the system is stable if:²³

$$\text{a) } 1 - 2m > 0; \text{ b) } L_r + L_w W_r < \min[m_4 W_r, m'_4 W_r]; \text{ and c) } g_3 > 0 \text{ or } g'_3 > 0$$

Thus, if the sum of the marginal propensities to import is less than unity and if the long run money demand function is inversely related to the interest rate, stability is insured as long as one country pursues fiscal stabilization policy.

Furthermore, even if sterilization rates differ, the system will be stable (if a, b, and c hold), provided $g_3 = g'_3$.²⁴ In this case, one characteristic root (that corresponding to the Balance of Trade), σ_1 , is the same as in (24); the other root becomes:

$$(25) \quad \bar{\sigma}_2 = \frac{-\alpha(1-t)g_3 [(m_4(1+m'_1) + m'_4(1+m_1) - L_w(2+m_1+m'_1))W_r - L_r(2+m_1+m'_1)]}{(2+m_1+m'_1)[\alpha W_r L_y(1+tg_3) - L_r(g_3(1-t) + s(1+tg_3))]}$$

From (25) it is apparent that even if one country fully sterilizes ($m'_1 = -1$), the system will still be stable provided $[(m'_4 - L_w)W_r - L_r] > 0$, $m_1 > -1$.

Though sterilization rates do alter stability in this case, they can either lower or increase the speeds of convergence. In particular:

$$(26) -\text{sign}\left[\frac{\partial \sigma_2}{\partial m_1}\right] = \text{sign}\left[(1 + m_1')(m_4' - m_4)\right]$$

If both countries monetize debt at the same rate, sterilization again is irrelevant. If $m_4 \neq m_4'$, then the country with the lower monetization rate should accomodate its Balance of Payments surplus (or deficit), whereas the other should sterilize.²⁵ The explanation for this result is that if there is deficient (excessive) aggregate demand, the country with the higher monetization rate will run a Balance of Payments deficit (surplus). If the country with the deficit (surplus) sterilizes and the surplus (deficit) country accommodates, the world money supply is increased (decreased), thus speeding convergence towards full employment equilibrium.

Thus, while sterilization may affect speeds of convergence if monetary policies differ, there is certainly no presumption that sterilization retards adjustment. Moreover, the preceding analysis implies that convergence is enhanced by financing government expenditures through money creation. When both governments finance their deficits via monetary issuance, monetary policy is impotent in altering the speed of adjustment of the balance of payments. That this should be so, even if sterilization rates differ, is a truly remarkable result.²⁶

Case II - Fiscal Policy to Income, Monetary Policy to the Balance of Trade

This assignment is one that cannot be discussed in small country models because of the one to one relationship between domestic income and the balance of trade. In terms of equations 20-23, this assignment can be represented by:

$$20b) (1 - t)G = g_3[Y^* - Y];$$

$$21b) (1 - t')G' = g'_3[Y'^* - Y']$$

$$22b) \dot{B}^c = m_2 \dot{B}T + m_4 \dot{B};$$

$$23b) \dot{B}'^c = m'_2 \dot{B}'T' + m'_4 \dot{B}'$$

From the results of the previous section, we intuitively expect monetary policy to be impotent when directed towards the Balance of Trade, as it is when directed towards the Balance of Payments. In fact, when corresponding government policy parameters are equal, the characteristic roots for the two assignments are identical!

Since the effects of various parameters on convergence have been discussed in detail, there is little point in repeating this exercise. Suffice it to say that convergence is enhanced by financing the government deficit through monetary issuance, rather than by bond issuance or taxes. It is also not surprising to find that monetary policy directed toward the balance of trade is impotent. As long as $m_2 = m'_2$, the world money supply is unaltered by such policy actions, hence the assignment of money to the balance of trade cannot have any lasting effect on the economic system.²⁷

Case III - Fiscal Policy to the Balance of Trade, Monetary Policy to Income

Again, this represents an assignment that is meaningful only in a multi-country model. As noted earlier, Balance of Trade disequilibria represent an improper distribution of the existing stock of wealth; in particular, a surplus implies a larger excess demand for wealth in the surplus country than the deficit country. Thus, it seems appropriate to apply fiscal policy to this target. Wealth, then, will be increased in the country with the balance of trade surplus and decreased in the country with the balance of trade deficit.

Also, aggregate unemployment represents an excess of wealth demand (at full employment) over the existing wealth stock; one way to accomodate

this problem is to increase wealth stocks (a policy pursued in the first two assignments). An alternative policy is to lessen wealth demand at full employment. Monetary policy can reduce the demand for wealth since increases in world money supplies lower interest rates, leading to decreased desires to hoard and thus increased spending. Therefore, unlike the previous assignments, we expect both tools to influence the stability of the system. In terms of equations 20-23, this assignment can be represented by:

$$20c) (1 - t)G = g_2 BT;$$

$$21c) (1 - t')G' = g_2' BT'$$

$$22c) \dot{B}^c = m_3[Y^* - Y] + m_4 \dot{B};$$

$$23c) \dot{B}'^c = m_3'[Y'^* - Y'] + m_4' \dot{B}'$$

From the Appendix it is readily seen that the characteristic roots for this assignment are:²⁸

$$(27) \sigma_1 = -2\alpha m[1 + (1-t)g_2][2m(1 + (1-t)g_2) + s(1-2m(1-tg_2))]^{-1}$$

$$\sigma_2 = -[\alpha W_r m_3][\alpha W_r L_y - sL_r]^{-1}$$

There are several noteworthy features of this assignment. First, note that both monetary and fiscal policy matter; fiscal policy (g_2) affects the convergence of the Balance of Trade, monetary policy affects aggregate demand (footnote 28). Moreover, the system is unambiguously stable; no condition on the long run money demand function or on the sum of the marginal propensities to import is needed. Finally, the system is stable even if $g_2 = 0$ or $t = 1$. The fact that it is stable for $g_2 = 0$ reflects the self-correcting nature of balance of trade disequilibria.

As in previous assignments, higher tax rates retard the speed of adjustment (for $g_2 > 0$); this occurs because higher tax rates decrease the impact of government purchases on the stock of wealth. However, as noted

above, the system is stable even under balanced budget financing because trade disequilibria tend to correct themselves, and because taxes, through lowering disposable income, decrease wealth demands, thereby partially offsetting the causes of Balance of Trade disequilibria.

Turning to fiscal expenditures (g_2), it is seen they have no impact on aggregate demand (σ_2); this is apparent since government expenditures are not directed towards this target and because, in the symmetric case, no net wealth is created. Thus, there can be no impact of fiscal policy on aggregate demand. The effect of fiscal policy on the Balance of Trade turns out to be ambiguous, depending on import propensities and tax rates. Thus:

$$(28) \text{ sign } \left[\frac{\partial(\sigma_1)}{\partial g_2} \right] = -\text{sign } [1 - 2m - t];$$

for $t = 0$, larger g_2 increases convergence if $2m < 1$. This follows from the fact that if $2m > 1$, U.K. government spending has a larger impact on U.S. income than does U.S. government spending. Thus the assignment of fiscal policy proves counterproductive. Further, since higher tax rates lower the efficacy of government spending, it follows that this assignment of fiscal policy can retard convergence if import propensities and tax rates are large.

The effects of monetary policy are straightforward; expansionary monetary policy (large m_3) is always stabilizing because of the effect of a change in the interest rate on wealth. Further, monetary policy has no impact on Balance of Trade adjustment (σ_1). As is intuitive, monetization of the debt (m_4) is irrelevant because no new wealth is created.

The case in which the magnitudes of policy responses differ presents no special problem in this assignment. Assuming only the same tax rates

in each country, the characteristic roots are readily calculated from the Appendix:

$$(29) \bar{\sigma}_1 = \frac{-2\alpha m \left[1 + (1-t) \left(\frac{g_2 + g_2'}{2} \right) \right]}{2m \left[1 + (1-t) \left(\frac{g_2 + g_2'}{2} \right) \right] s \left[1 - 2m \left(1 - t \left(\frac{g_2 + g_2'}{2} \right) \right) \right]}$$

$$\bar{\sigma}_2 = -\alpha W_r (m_3 + m_3') [2(\alpha W_r L_y - s L_r)]^{-1}$$

The close relation between (29) and (27) is apparent, and thus it is not necessary to repeat the analysis of the effects of various policies on stability. We just note that stability remains assured and that debt monetization does not matter. This latter follows because, even though there may be wealth creation in the asymmetric case, there is no relation between aggregate demand equilibrium and Balance of Trade equilibrium.

To conclude, this assignment is, in some sense, the most stable since no assumptions are needed on the long run money demand function or marginal propensities to import. Further, both instruments affect convergence in this assignment. As in previous assignments, it is inappropriate to couple taxes with stabilizing government expenditures. However, the impact of fiscal policy on stability depends on the sum of the marginal propensities to import (for $t = 0$); if no presumption exists as to the magnitudes of these parameters, no conclusions emerge as to the efficacy of fiscal policy. (Note that for previous assignments instability could result if $2m > 1$).

Case IV - Monetary Policy to Income, Fiscal Policy to the Balance of Payments

This assignment, which is the anti-Mundell assignment, seems the most likely to be unstable. While monetary policy is coupled with a target for which it is effective, fiscal policy is not so paired. To the extent a

Balance of Payments surplus is due to transitory capital flows, expansionary fiscal policy can be destabilizing as an increase (decrease) in the stock of bonds in the surplus (deficit) country will increase (decrease) the demand for money, aggravating the balance of payments.

This policy can be represented by:

$$20d) (1 - t)G = g_1 BP;$$

$$21d) (1 - t')G' = g'_1 BP$$

$$22d) \dot{B}^C = m_3[Y^* - Y] = m_4 \dot{B}$$

$$23d) \dot{B}'^C = m'_3[Y'^* - Y'] + m'_4 \dot{B}'$$

For this assignment there are three characteristic roots;²⁹ the one corresponding to income levels is:

$$30) \sigma_1 = -\alpha W_r m_3 [\alpha W_r L_y - s L_r]^{-1} < 0$$

which means aggregate demand converges towards equilibrium. The other two roots are the solutions to the following quadratic:

$$\begin{aligned} 31) \sigma^2 L_y g_1 [1 - t - 2m] + \sigma \left[(1 - t - 2m)(\alpha L_y g_1 + s L_w g_1) + \right. \\ \left. (1 - t + st)g_1 m_3 (1 - 2m) - \left\{ (1 + g_1 (1 - t)m_4)(2m + s(1 - 2m)) \right\} \right] \\ \left. + \alpha[(1 - 2m)(1 - t)g_1 m_3 - 2m(1 + (1 - t)g_1 m_4)] = 0. \right. \end{aligned}$$

For $g_1 = 0$, the quadratic is degenerate, and the system is stable; the root obtained is identical to that for $g_2 = 0$ in the previous assignment. However, for $g_1 \neq 0$, the stability of the system is in doubt; it depends upon the underlying structural parameters, as well as the government response parameters.³⁰

Mixed Assignments

Thus far we have concentrated upon pure assignments, in which an instrument is paired with a single target. However, mixed assignments are also

easily investigated because of the assumed linearity of the system around equilibrium. As we have just shown, the assignment of fiscal policy to the Balance of Payments can introduce instability into the system. However, other mixed assignments will, in general, be stable. The characteristic roots for the mixed policy assignment, under the assumption $g_1 = g_1' = 0$, and government response parameters are the same in each country ($g_i = g_i'$; $m_i = m_i'$) are:³¹

$$37) \sigma_1 = \frac{-[2\alpha m + 2\alpha m(1 - t)g_2 + \alpha(1 - 2m)(1 - t)g_3]}{[s(1 - 2m) + 2m + 2mg_2(1 - t + st) + g_3(1 - 2m)(1 - t + st)]}$$

$$\sigma_2 = \frac{-\alpha[m_3 W_r + (1 - t)g_3 \{W_r(m_4 - L_w) - L_r\}]}{[\alpha W_r L_y (1 + tg_3) - L_r(s + (1 - t + st)g_3)]}$$

The properties discussed previously hold for the mixed assignment. Some of the more important results can be summarized as follows: a) stability is assured if $g_3 = 0$ and $m_3 > 0$; b) monetary policy assigned to the Balance of Trade or Payments has no impact on speeds of convergence; c) financing fiscal stabilization policy by taxes retards convergence; d) if fiscal policy is used to stabilize aggregate demand, these expenditures should be financed via money creation (or contraction); e) monetary policy only affects aggregate demand; because of perfect capital markets it has no effect on the stability and convergence of the Balance of Trade or Payments; f) larger rates of fiscal policy towards aggregate demand increases speed of convergence towards aggregate demand equilibrium if $(W_r(m_4 - L_w) - L_r) > 0$, and increases the speed of convergence towards Balance of Trade equilibrium if $(1 - 2m)(1 - 2m - t) > 0$; g) fiscal policy towards the Balance of Trade has no effect on aggregate demand, but increases in g_2 increase convergence towards Balance

of Trade equilibrium if $(1 - 2m - t) > 0$; and h) increases in m_3 always increase the speed of convergence towards aggregate demand equilibrium. All of these results can be viewed as a synthesis of the properties of the pure assignment cases.

Section IV - Conclusion

In this paper we have explored the assignment problem in a two country model in which the demand for wealth, as well as the portfolio composition of wealth, represent stock, rather than flow decisions. In small country versions of this model (as well as in Keynesian models, if it is not possible to sustain Balance of Trade disequilibria over the long run) both monetary and fiscal policy are impotent in affecting the equilibrium income level because stock equilibrium requires Balance of Trade equilibrium. Thus, in such small country models expenditure switching policies (exchange rate changes, tariffs, differential government propensities to import) are required to alter domestic income.

However, we feel that it is more appropriate, particularly for the developed countries of the Western world, to investigate such issues in an interdependent setting. In this context we have found that either monetary or fiscal policy can alter income levels and that, because of the inherent adjustments caused by Balance of Trade or Payments disequilibria, only one target (aggregate demand) does not necessarily converge to the target level. In investigating various assignments we have shown that, provided some instrument is assigned to income, the system tends to be stable, if fiscal policy is not assigned to Balance of Payments equilibrium. While it is difficult to state which assignment is "best", since this entails comparing rates of expansion of the money supply to rates of government spending, in

some sense the most appropriate assignment appears to be to assign fiscal policy to Balance of Trade disequilibria and monetary policy to income. We say that this is "most appropriate" both because it represents an assignment of instruments according to the causes of disequilibria, and because it is stable under a wider range of values for the underlying parameters.

To the extent that we adopt this latter assignment, our conclusions differ from Mundell's for the obvious reason that we view capital movements as stock adjustments, rather than sustainable flows. On the other hand, our results are consistent with those of Tsiang (1975, 207). In his paper, Tsiang argues that fiscal policy should be assigned to Balance of Payments equilibrium, once it is purged of transitory capital flows. However, since he does not view wealth holdings as a stock decision, his model allows wealth stocks to permanently grow, even though real income does not. If no sustainable capital flows are possible (as is the case if wealth stocks do not grow), then his assignment reduces to ours. Of course, it should be noted that, even in the context of his own model, Tsiang provides no suggestions as to how policy makers can distinguish "volatile" transitory capital flows from what he perceives as sustainable flows.

Perhaps our most interesting results concern the use of monetary policy. Mundell argues that monetary policy should be assigned to the Balance of Payments; this follows from his flow specification of the model. Other authors (Aliber 1974, Argy and Kouri 1974), even those working implicitly in the context of stock adjustment models have argued that sterilization retards adjustment, and that policy makers should accomodate Balance of Payments disequilibria. Tsiang, (1975, 211) on the other hand, argues that "it would seem advisable to sever... the link between domestic money

supply and the balance of payments." However, these results have all been developed in the context of one country models. In our paper we find that sterilization policy, provided both countries do not fully sterilize, has no impact on the speed of adjustment towards equilibrium.

All of our analysis has been conducted in the context of perfect capital markets; it would seem that a logical extension of this work would be to consider how "imperfections" in international capital markets affect these results. While many models have assumed imperfect capital mobility, little explanation has been given for the cause of this imperfection, and the degree of capital mobility has been taken as exogenous. Since it seems that such imperfections arise due to risks perceived by investors (such as exchange risk), and since the risk could well be treated as an endogenous variable (say, as reflected in international reserve holdings), we are currently attempting to expand this model to incorporate endogenous risk and imperfect capital mobility into it. We feel that this represents an appropriate vehicle for the study of international problems in an interdependent world.

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$$10a) \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} y - y' \\ y + y' \\ R \end{bmatrix} \quad \text{Where: } y \equiv -(Y^* - Y) \\ y' = -(Y'^* - Y') \\ R = \text{U.S. reserves}$$

Note that $y(y')$ represents the deviation of actual from desired income and that exchange rates are compatible with full employment (see footnote 17) so that $mY^* = m'Y'^*$. The sum of the deviations $(y + y')$ measures the state of world inflationary or recessionary pressure. Since the composition of world wealth will be determined by differences in income levels, $y - y'$ is a proxy for the balance of trade. The above follows as the balance of trade will be zero if $y = y'$; for there to be no inflationary or recessionary pressure $y + y'$ must equal zero.

The nine elements of the matrix 10a) are as follows:

$$\begin{aligned}
 \text{(i)} \quad a_{11} = & \sigma L_y \left[1 + \frac{tm}{1-2m} (g_2 + g_2') + \frac{t}{2} (g_3 + g_3') \right] + \left(\frac{m_3 + m_3'}{2} \right) \\
 & + \left(\frac{m}{1-2m} \right) \left[(m_2 + m_2') - 2L_w + (1-t)(g_2(m_4 - L_w) + g_2'(m_4' - L_w)) \right] \\
 & + \frac{(1-t)}{2} \left[g_3(m_4 - L_w) + g_3'(m_4' - L_w) \right]
 \end{aligned}$$

$$\text{(ii)} \quad a_{12} = \frac{\sigma L_y t}{2} [g_3 - g_3'] + \left[\frac{m_3 - m_3'}{2} \right] + \left(\frac{1-t}{2} \right) [g_3(m_4 - L_w) - g_3'(m_4' - L_w)]$$

$$\text{(iii)} \quad a_{13} = -\sigma L_y t (g_1 + g_1') - 2 - (m_1 + m_1') - (1-t) [g_1(m_4 - L_w) + g_1'(m_4' - L_w)]$$

$$\begin{aligned}
 \text{(iv)} \quad a_{21} = & \sigma \left[s + \frac{2m}{1-2m} + \frac{(1-t+st)m(g_2 + g_2')}{(1-2m)} + \frac{(1-t+st)(g_3 + g_3')}{2} \right] \\
 & + \alpha \left[\frac{2m}{1-2m} + \frac{m(1-t)(g_2 + g_2')}{1-2m} + \frac{(1-t)(g_3 + g_3')}{2} \right]
 \end{aligned}$$

$$\text{(v)} \quad a_{22} = \frac{\sigma(1-t+st)}{2} (g_3 - g_3') + \frac{\alpha(1-t)}{2} (g_3 - g_3')$$

$$\text{(vi)} \quad a_{23} = -[\sigma(1-t+st) + \alpha(1-t)](g_1 + g_1')$$

$$\begin{aligned}
 \text{(vii)} \quad a_{31} = & -\sigma(\alpha W_r t L_y - L_r(1-t+st)) \left(\frac{m(g_2 - g_2')}{1-2m} - \frac{(g_3 - g_3')}{2} \right) \\
 & - \frac{\alpha m}{1-2m} \left[(m_2 - m_2') W_r - (1-t) L_r (m_2 - m_2') + (1-t) W_r \{ g_2 (m_4 - L_w) - g_2' (m_4' - L_w) \} \right] \\
 & - \frac{\alpha}{2} \left[W_r (m_3 - m_3') - L_r (1-t) (g_3 - g_3') + W_r (1-t) \{ g_3 (m_4 - L_w) - g_3' (m_4' - L_w) \} \right]
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad a_{32} = & \sigma \left[s L_r - \alpha W_r L_y - \left(\frac{g_3 + g_3'}{2} \right) (\alpha W_r t L_y - (1-t+st) L_r) \right] \\
 & - \frac{\alpha}{2} \left[W_r (m_3 + m_3') - (1-t) L_r (g_3 + g_3') + (1-t) W_r \{ g_3 (m_4 - L_w) - g_3' (m_4' - L_w) \} \right]
 \end{aligned}$$

$$\begin{aligned}
 \text{(ix)} \quad a_{33} = & \sigma(g_1 - g_1') (\alpha W_r t L_y - (1-t+st) L_r) \\
 & + \alpha \left[W_r (m_1 - m_1') - (1-t) L_r (g_1 - g_1') + (1-t) W_r \{ g_1 (m_4 - L_w) - g_1' (m_4' - L_w) \} \right]
 \end{aligned}$$

Section II contained a discussion of the stability of the system when governments set all $g_i = g_i' = m_i = m_i' = 0$. This discussion did not assume identical parameters of corresponding U.S. and U.K. asset demand functions, and the derivation of these results is given in Enders (1975). In the symmetric case the sole root for the system is:

$\sigma_1 = -2\alpha m[s(1-2m) + 2m]^{-1}$ which is unambiguously negative as the marginal propensity to save is less than unity. Returning to the case in which governments are policy active, note that if $2m = 1$, the system is degenerate. The symmetry of the model--for $m = 1/2$ --implies $Y = Y'$ so $y' - y$ is identically zero.

The characteristic roots presented in the text are obtained from the characteristic equation of 10a. Note that this specification allows us to consider mixed assignments. The symmetric case ($g_i = g_i', m_i = m_i'$) in which $g_1 = g_1' = 0$ (fiscal policy is not assigned to the Balance of Payments) is particularly simple since $a_{12} = a_{22} = a_{23} = a_{31} = a_{33} = 0$. In this case the characteristic equation is a quadratic (if $m_i = m_i' \neq -1$) and the roots are obtained from $a_{21} = 0$; $a_{32} = 0$. Moreover, the root from a_{21} corresponds to $y' - y$ (BT) and $R(BP)$, whereas the root for a_{32} corresponds to $(y' + y)$ i.e., aggregate demand. All of our results in section III come from solving the characteristic equation (and the associated characteristic roots and vectors) from 10a) in both symmetric and asymmetric cases.

Footnotes

1. Fostin (1975) discusses the breakdown of this principle in the n target, n instrument case.
2. See, for example, Tobin (1969), Tobin and Brainard (1968), or Markowitz (1959).
3. See, for example, Levin (1972), Patrick (1968), Floyd (1972) or Roper (1971).
4. This point is made by Aghevli and Borts (1974), and implied by Johnson (1972). The reader should note the obvious similarity between these recent arguments and the Hume specie-flow mechanism.
5. One of the first authors to stress the importance of this point was Johnson (1961). It is important to distinguish between the wealth increasing nature of a balance of trade surplus and the asset substitution effect of a balance of payments surplus. The confusion between the two is exemplified by Mussa (1974, 338) who states: "The monetary approach emphasizes that as a fact of accounting the balance of payments surplus is identically equal to the excess of income over expenditure."
6. See, for example, Johnson (1972), Boyer (1975), Girton and Henderson (1975), Frenkel and Rodriquez (1975) Komiya (1969), or Mussa (1974).
7. Jones (1968).
8. Girton and Henderson (1975).
9. See, Girton and Henderson (1975) or Boyer (1975).
10. As the life cycle and permanent income hypotheses imply individuals desire to hold a terminal stock of wealth. See Modigliani and Brumberg (1954) or Friedman (1957).
11. Many authors, such as Tsiang, seem to ignore considerations of how government purchases are financed, and thus miss the wealth creation effects of government deficits and the problem as to how such deficits should be financed.
12. Mathieson (1975) demonstrated that by controlling the required reserve ratio, the monetary authorities, in a small country, could gain a degree of monetary control. Aside from the institutional constraints and the reluctance of central banks to alter reserve ratios, this finding is not relevant to the Assignment Problem. As fiscal policy cannot alter the level of income, the reserve ratio necessarily must be assigned to internal balance in a small country.

13. The model is easily adaptable to allow residents of each country to hold two monies.
14. Johnson(1972) stated that the monetary approach to the balance of payments necessarily assumes full employment and flexible prices. Mussa (1975), Argy and Kouri (1974) and McKinnon (1968), however, used monetary models with fixed prices and variable income levels.
15. This assumption can be relaxed and each government's marginal propensity to import can be treated as a policy instrument. However, the assumption used in the text is consistent with much of the work concerning the effects of tariffs on the terms of trade and can be rationalized by assuming residents consider government purchases in making their own consumption decisions.
16. The difficulties involved with incorporating interest payments are well known to macro-theorists. It is not our intent to tackle these problems here. Rather we assume that each government sterilizes the international interest payments, via lump sum taxes. Interest payments, then do not appear in equation 15 nor do they add to disposable income. Obviously, it must be assumed that the amount of tax any individual pays is not commensurate with that individuals holdings of bonds. Readers interested in this point are referred to Levin (1972) and Tsiang (1975), although we do not believe that these authors have handled this problem successfully.
17. We assume that the fixed exchange rate is such that the desired income levels are consistent with steady state equilibrium. This assumption must be made in any model where sustainable capital flows are not viable. Since the underlying structural parameters are not known it is an interesting, but separate, issue to determine when governments should adjust exchange rates, assuming such adjustments are costly.
18. The condition that $1-2m > 0$ is a familiar one from the literature on the transfer problem, and the secondary burden of a transfer, if prices are flexible. It arises in this model because imports depend on expenditures, not income.
19. It is difficult to compare speeds of adjustment for different assignments since this entails comparing response parameters of monetary and fiscal policy.
20. Subscripts denote partial differentiation, e.g.

$$\frac{\partial L}{\partial y} = L_y; \frac{\partial L}{\partial r} = L_r; \frac{\partial W^*}{\partial r} = W_r.$$

21. Note that if $2m = 1$, g_3 does not affect σ_1 . As shown in the appendix, when $g_1 = g'_1$ and $m_1 = m'_1$, it is possible to partition the system. In this case σ_1 is the characteristic root corresponding to the balance of trade and payments, and σ_2 corresponds to income levels. If $2m = 1$, g_3 does not affect σ_1 , or the balance of trade or payments. If $t = 1$, g_3 does not affect σ_2 , or the level of income.

22. Sterilizing or accommodating the balance of payments will, however, affect the equilibrium distribution of reserves, see Enders (1975). Our result in the text follows from allowing $m_1 = m'_1$. Under this assumption the world money supply is fixed, i.e., the increase in the world money supply due to sterilization is $m_1 BP + m'_1 BP'$. Since $m_1 = m'_1$ and $BP = -BP'$, the world money supply is fixed. The case in which $m_1 \neq m'_1$ is discussed below.
23. For this case, the characteristic equation of the system is rather complicated, and the roots are not easily simplified. These roots are readily obtained from the Appendix and are omitted here to save space. The conditions shown in the text can, however, be shown to be sufficient conditions for stability by examining the coefficients of the quadratic. Note that for $m_4 = m'_4 = 0$, conditions a, b and c reduce to the earlier conditions for stability. Also note that if only one country responds to unemployment (i.e., either g_3 or $g'_3 = 0$), the system still converges to the full employment income level for each country.
24. If all policies differ, the model becomes quite complex. It is possible to show that stability will hold; but comparative statics results are difficult to obtain in this case.
25. Note that if one country fully sterilizes, the other country's monetary policy towards the Balance of Payments is irrelevant.
26. Since monetary policy to the Balance of Payments affects only the convergence towards full employment, and not the Balance of Trade or Payments, sterilization is irrelevant in this case. This follows from the absence of a connection between the state of world aggregate demand and the Balance of Payments position of each country. Note that this assumes that $m_4 = m'_4$.
27. Even if policy response parameters differ the system will, in general, be stable; however, the general case is quite complex, and the characteristic roots are irrational. If fiscal policy response parameters are the same ($g_3 = g'_3$), the quadratic is readily solvable; one root is the same as σ_1 in (24); the other is:

$$\sigma_2 = \frac{-\alpha(1-t)g_3 \left[W_r \left\{ \left(\frac{m_4 + m'_4}{2} \right) - L_w \right\} - L_r \right]}{[\alpha W_r (1+tg_3) - L_r (g_3(1-t) + s(1+tg_3))]}$$

Again, it is clear full monetization of the debt promotes stability, but that otherwise monetary policy is impotent (even for $m_2 \neq m'_2$).

28. As in previous assignments, σ_1 corresponds to the speed of convergence of the Balance of Trade and Payments, whereas σ_2 corresponds to convergence of aggregate demand. This is seen by considering the eigenvector corresponding to each root.
29. These roots can be obtained from the Appendix. Note that previous assignments yielded degenerate cubic characteristic equations in that these were only two roots. From the eigenvector, it can be seen that the roots of 31 correspond to the Balance of Trade and Payments account.

30. The case of asymmetric government response parameters is even more complicated, and little can be said about the stability of the system. However, the uncertainty attached to the properties of this assignment, and the inability to conclude that this assignment is stable is sufficient to allow us to conclude, as Mundell does, that such an assignment is inappropriate.
31. The asymmetric mixed assignment will, in general, yield a cubic equation that is difficult to analyze. These roots, again, can be obtained from the Appendix. As before, σ_1 corresponds to the Balance of Trade and Payments and σ_2 to aggregate demand.

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